4-3-2013

Cylindrical Co-ordinates

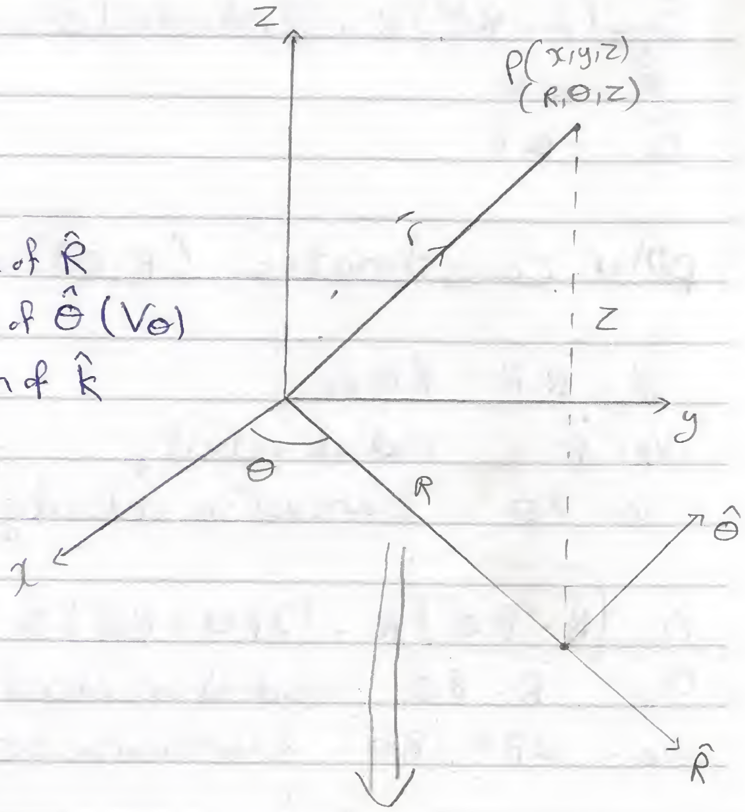
$$\bar r = R\hat R + z\hat k$$

$$\frac{d\bar r}{dt} = \bar V = \dot R\hat R + R\dot{\hat R} + \dot z\hat k$$

$\dot R\hat R \Rightarrow V_R \Rightarrow$ Velocity in direction of $\hat R$

$R\dot{\hat R} = R\dot\theta\hat\theta \Rightarrow$ Velocity in direction of $\hat\theta$ (V_θ)

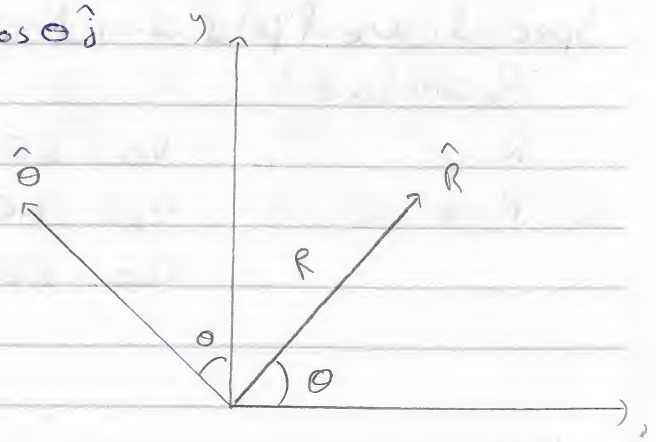
$\dot z\hat k \Rightarrow V_z \Rightarrow$ Velocity in direction of $\hat k$



$\hat R = \cos\theta\,\hat i + \sin\theta\,\hat j$, $\hat\theta = -\sin\theta\,\hat i + \cos\theta\,\hat j$

$$\dot{\hat R} = -\sin\theta(\dot\theta)\hat i + \cos\theta(\dot\theta)\hat j$$
$$= \dot\theta(-\sin\theta\,\hat i + \cos\theta\,\hat j)$$
$$\dot{\hat R} = \dot\theta\hat\theta$$

$$\dot{\hat\theta} = -\cos\theta(\dot\theta)\hat i - \sin\theta(\dot\theta)\hat j$$
$$= -\dot\theta(\cos\theta\,\hat i + \sin\theta\,\hat j)$$
$$\dot{\hat\theta} = -\dot\theta\hat R$$



$$\bar V = \dot R\hat R + R\dot\theta\hat\theta + \dot z\hat k$$

$$\frac{d\bar V}{dt} = \bar a = \ddot R\hat R + \dot R\underbrace{\dot{\hat R}}_{\dot\theta\hat\theta} + \dot R\dot\theta\hat\theta + R\ddot\theta\hat\theta + R\dot\theta\underbrace{\dot{\hat\theta}}_{-\dot\theta\hat R} + \ddot z\hat k$$

$$\bar a = (\ddot R - R\dot\theta^2)\hat R + (2\dot R\dot\theta + R\ddot\theta)\hat\theta + \ddot z\hat k$$

Summary

$\bar{r} = R\hat{R} + Z\hat{k}$

$\bar{V} = \dot{R}\hat{R} + R\dot{\theta}\hat{\theta} + \dot{Z}\hat{k}$

$\bar{a} = (\ddot{R} - R\dot{\theta}^2)\hat{R} + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{\theta} + \ddot{Z}\hat{k}$

$\dot{\hat{R}} = \dot{\theta}\hat{\theta}$

$\dot{\hat{\theta}} = -\dot{\theta}\hat{R}$

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polar co-ordinates (R, θ)

$V = \dot{R}\hat{R} + R\dot{\theta}\hat{\theta}$

$V_R = \dot{R}$ radial velocity

$V_\theta = R\dot{\theta}$ transverse velocity

$a = (\ddot{R} - R\dot{\theta}^2)\hat{R} + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{\theta}$

$a_R = \ddot{R} - R\dot{\theta}^2$ radial acceleration

$a_\theta = 2\dot{R}\dot{\theta} + R\ddot{\theta}$ transverse acceleration

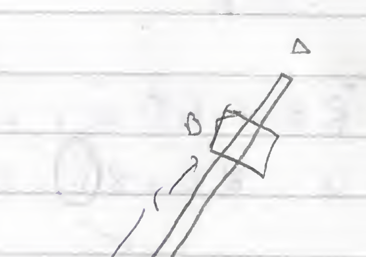
Special case of polar co-ordinates: circular motion

R constant

$\dot{R} = 0$ $\quad V_\theta = R\dot{\theta}$

$\ddot{R} = 0$ $\quad a_R = -R\dot{\theta}^2$, $\cancel{a_\theta = \ddot{z}\hat{R}}$

$\quad a_\theta = R\ddot{\theta}\hat{\theta}$



Ex. 11.12

$\theta = 0.15t^2$, $r = 0.9 - 0.12t^2$ at $\theta = 30^\circ$ $OA = 0.9\,m$

a) $\dot{r} = -0.24t$, $\ddot{r} = -0.24$

$\dot{\theta} = 0.3t$, $\ddot{\theta} = 0.3$

$\theta = 30^\circ = \frac{\pi}{6}$ $\quad \frac{\pi}{6} = 0.15t^2$ $\quad t = 1.87$

$V = \sqrt{V_R^2 + V_\theta^2}$

$= \sqrt{(\dot{r})^2 + (R\dot{\theta})^2} = \sqrt{(-0.24 \times 1.87)^2 + (0.3 \times 1.87 \times 0.48)^2}$

$= 0.52$

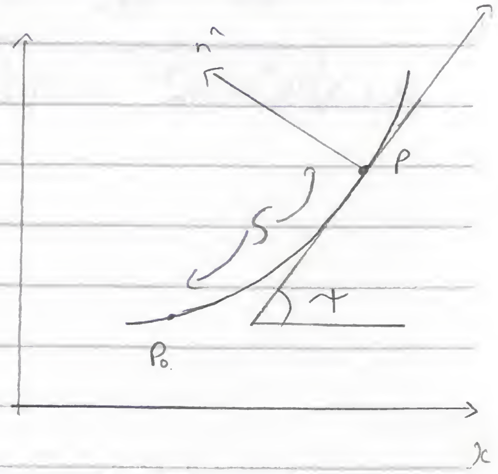
b) ~~$\bar{a} = \ddot{R} - R\dot{\theta}^2$~~

b) $a = \sqrt{a_R^2 + a_\theta^2} = \sqrt{(-0.24 - 0.48 \times 0.56^2)^2 + (0.48 \times 0.3 + 2 \times -0.449 \times 0.56)^2}$

$= 0.531\ m/s^2$

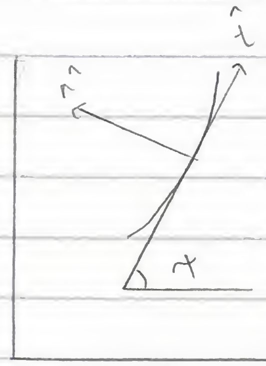
c) $a_{B/OA} = -0.24\ m/s^2$ ($0.24\ m/s^2$ towards O)

Intrinsic Co-ordinates



$\bar{V} = v\hat{t} = \dot{s}\hat{t}$

$\bar{a} = \dfrac{d\bar{v}}{dt} = \ddot{s}\hat{t} + \dot{s}\dot{\hat{t}}$



$\dot{\hat{t}} = \dot{\psi}\hat{n}$ (same as $\dot{\hat{R}} = \dot{\theta}\hat{\theta}$)

$\dot{\psi} = \dfrac{d\psi}{dt} = \dfrac{d\psi}{ds} \cdot \dfrac{ds}{dt}$ $\qquad \dfrac{ds}{d\psi} = \rho \quad \therefore \dfrac{d\psi}{ds} = \dfrac{1}{\rho}$

$\dfrac{ds}{dt} = \dot{s}$ $\qquad \dot{\psi} = \dfrac{\dot{s}}{\rho}$ $\qquad \dot{\hat{t}} = \dfrac{\dot{s}}{\rho}\hat{n}$ $\qquad \rho$ = radius of curvature

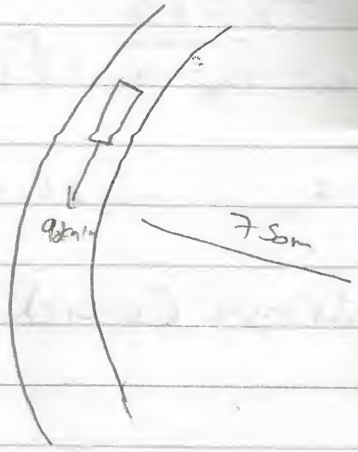
$\bar{a} = \ddot{s}\hat{t} + \dot{s}\dot{\hat{t}}$

$= \ddot{s}\hat{t} + \dfrac{\dot{s}^2}{\rho}\hat{n}$

$\ddot{s}$ → tangential acceleration a_t

$\dfrac{\dot{s}^2}{\rho}$ → normal acceleration a_n

Ex 11.10



$90\ km/h = 25\ m/s$

$72\ km/h = 20\ m/s$

$$\bar{a} = a_t \hat{t} + a_n \hat{n} = \dot{v}\hat{t} + \frac{v^2}{\rho}\hat{n}$$

$$a_t = \frac{v-u}{t} = \frac{20-25}{8} = \frac{-5}{8}\ m/s^2$$

$$a_n = \frac{V^2}{\rho} = \frac{25^2}{750} = \frac{5}{6}\ m/s^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{\left(-\frac{5}{8}\right)^2 + \left(\frac{5}{6}\right)^2} = 1.04$$